Turbulent kinetic energy budget in the far field of a square cylinder wake

S. L. Tang, N. Lefeuvre, L. Djenidi, R. A. Antonia & Y. Zhou

Abstract In this paper, the transport equation for the turbulent kinetic energy (TKE) is tested in the far field of a square cylinder wake (hereafter denoted SC). It is found that the physical mechanism for transporting TKE along the axis is different from that in a circular cylinder (hereafter denoted CC) wake. For the SC wake, the $\overline{q^2}$ diffusion term is negligible compared to advection term along the axis and the advection and energy dissipation terms dominate the budget. However, in the CC wake, aside from the advection and energy dissipation terms, the $\overline{q^2}$ diffusion term also contributes significantly to the budget. At the region close to the centreline, the gain of the energy due to the contributions from the advection and diffusion terms is equal to the loss due to the isotropic dissipation, indicating that the isotropic dissipation rate $\overline{\varepsilon}_{iso}$ is a good surrogate of the mean TKE dissipation rate $\bar{\varepsilon}$.

1 Introduction

The transport equation for the average kinetic energy ($\overline{q^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$, where u, v, and w are the velocity fluctuations in the x, y, and z directions, and overbar denotes time averaging) in the far field of a plane wake can be approximated to [1].

$$\underbrace{\frac{1}{2}U\frac{\partial \overline{q^2}}{\partial x}}_{advection} + \underbrace{\overline{uv}\frac{\partial U}{\partial y}}_{production} + \underbrace{\frac{\partial}{\partial y}(\frac{1}{2}\overline{vq^2} + \bar{p})}_{diffusion} + \underbrace{\bar{\varepsilon}}_{dissipation} = 0 \tag{1}$$

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U is the mean velocity in the streamwise direction and p is the pressure; $\overline{\varepsilon}$ is the mean TKE dissipation rate defined as

$$\bar{\varepsilon} = \frac{\nu}{2}\overline{(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})^2} \qquad (i,j = 1,2,3) \tag{2}$$

where ν is the kinematic viscosity. Several attempts have been made to measure the various terms of Eq. (1) in a CC. For example, Browne et. al. [1], Aronson and Lofdahl [2], and Lefeuvre et. al. [3] showed that, on the centreline of a CC, the advection, dissipation and $\overline{q^2}$ diffusion terms in Eq. (1) contribute significantly to the budget. In the region away from the centreline, the production term in Eq. (1) arises. It should be noted that there is a strong anisotropy in the far wake of a CC, even on the flow centreline where the mean shear is zero (e.g. Refs. [1, 4]). To our knowledge, there is no previous attempt to test the Eq. (1) in the far wake of a SC, although Lefeuvre (DNS data, private communication) recently observed that the advection term is balanced by the dissipation term in the intermediate wake of the SC up to $x/d = 100$. In the present investigation, Eq. (1) is tested, on the basis of X-wire measurements, in the far wake of a SC, with special attention paid to the surrogate of $\bar{\varepsilon}$.

2 Experimental details

X-wire Measurements were carried out in an open-return low-turbulence wind tunnel with a 2.2m long working section (0.35 m $\times$ 0.35 m). A circular cylinder of d=6.3 mm in diameter was installed in the midplane and the measurement stations are at $x/d = 230$ and 270. Experiments were carried out at a free-stream velocity U_∞=9.5m/s, corresponding to a Reynolds number $Re = U_\infty d/\nu$ = 4000.

The Wollaston (Pt-10%Rh) hot wires (diameter d_w=2.5 μm) were etched to an active length of 0.45-0.5 mm. The length to diameter ratio of the wires was typically 200. The hot wires were operated with constant-temperature anemometers at an overheat ratio of 1.5. The output signals from the anemometers were passed through buck and gain circuits and low-pass filtered (the cut-off frequency f_c was set close to the Kolmogorov frequency f_η). The signals were then digitized into a personal computer using a 12 bit A/D converter at a sampling frequency of 20 kHz. The record duration was about 100 s.

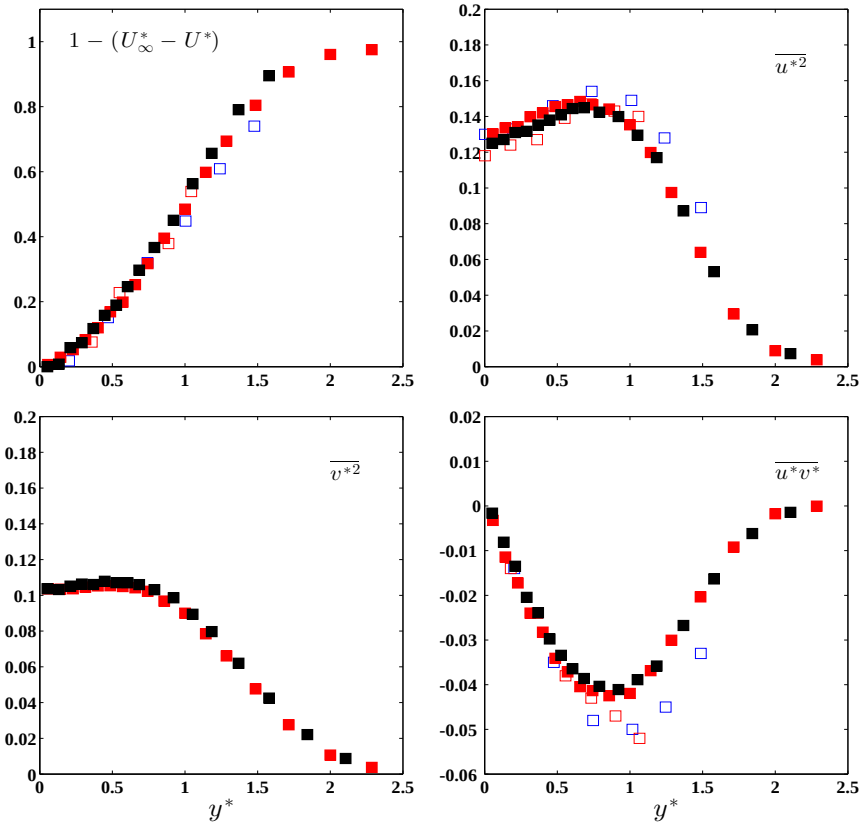


Fig. 1 Normalized mean velocity defect and Reynolds stresses; present data: $x/d = 230$, ■; $x/d = 270$, ■; Zhou and Antonia [5]: $x/d = 165$, ❑; $x/d = 375$, ❑.

3 Results

3.1 Test of self-preservation

Distributions of $1-(U^*_\infty - U^*)$, $\overline{u^{*2}}$, $\overline{v^{*2}}$, and $\overline{u^*v^*}$ are shown in Fig. 1. An asterisk denotes normalisation by the maximum velocity deficit U_d and the half-width L. Also shown are the data of Zhou and Antonia [5] at $x/d = 165$ and 375 for SC wake. There is reasonable collapse of the distributions for $1-(U^*_\infty - U^*)$. As pointed out by Zhou and Antonia [5], the distributions for $1-(U^*_\infty - U^*)$ are approximately independent of the wake generators in the far-wake region of the plane wakes, although discernible differences can be observed in the Reynolds stresses, spectra of v, and vorticities. In the case of $\overline{u^{*2}}$, there is a good collapse for the present data, and those of Zhou and Antonia [5]. The distributions for $\overline{v^{*2}}$ and $\overline{u^*v^*}$ also show a good collapse for the present measurements. The difference in the distributions

for $\overline{u^*v^*}$ between the present data and those of Zhou and Antonia [5] in the range $y^* = 1 \sim 1.5$ may due to the different initial conditions between two experiments. Note that the distributions of $\overline{v^{*2}}$ for Zhou and Antonia [5] are not shown since we suspect that they inadvertently used $\overline{u^{*2}}$ for $\overline{v^{*2}}$ in their Fig. 2c. Nevertheless, Fig. 1 suggests that self-preservation is well satisfied, at least in the context of Reynolds stresses, the far wake of a SC. This of particular interest in the study of far wake because it will allow the development of a theoretical analysis.

3.2 TKE budget of $\overline{q^2}$

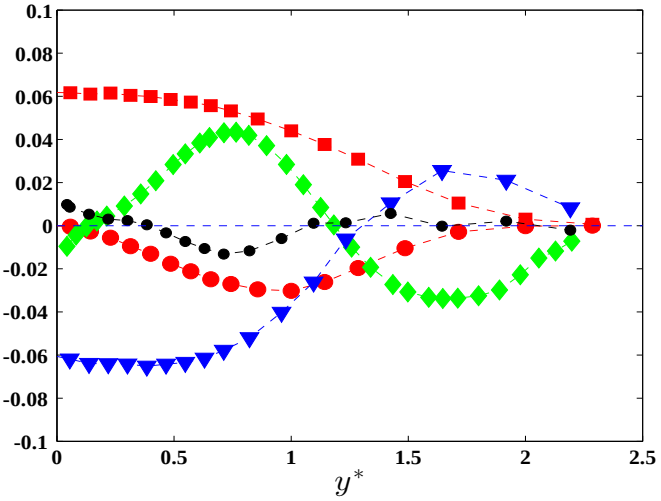


Fig. 2 Measured budget of average turbulent energyat $x/d = 270$. ■, isotropic dissipation; ◆, $\overline{q^2}$ diffusion; ▼, advection; ●, production; ●, pressure diffusion (by difference). Values normalized by L/U_d^3.

Fig. 2 shows all terms of Eq. (1), normalized by L/U_d^3. The mean TKE dissipation rate $\overline{\varepsilon}$ is estimated by assuming isotropy, $\overline{\varepsilon} = \overline{\varepsilon}_{iso} = 15\nu\overline{(\partial u/\partial x)^2}$. The pressure diffusion term is obtained by difference. In the region close to the wake centreline, *i.e.* $y^* = 0 \sim 0.3$, the gain of the energy due to the contributions from the advection is approximately equal to the loss due to dissipation. The $\overline{q^2}$ diffusion term, however, is negligible (the ratio of the $\overline{q^2}$ diffusion term to advection term is about 1/7 on the flow centerline). This is different from that in a CC wake, where, aside from the advection and energy dissipation terms, the diffusion term also contributes significantly to the budget (the ratio of the diffusion term to advection term is about 1 (*e.g.*

Refs. [1, 2, 3]). As the distance from the axis increases, both the $\overline{q^2}$ diffusion and production terms increase quickly and contribute significantly to the TKE budget. It is interesting to note that, in the region far from the wake centreline $y^* = 1.6 \sim 2.5$, both the production and dissipation terms are negligible but the $\overline{q^2}$ diffusion and advection terms contribute to the budget (they are approximately equal and of opposite sign). While the pressure diffusion term is small compared to the $\overline{q^2}$ diffusion term, they both satisfy the integral constraints by assuming symmetry with respect to $y^* = 0$, *i.e.* $\int_{-\infty}^{\infty} \frac{\partial \overline{vq^2}}{\partial y} dy = 0$ and $\int_{-\infty}^{\infty} \frac{\partial \overline{vp}}{\partial y} dy = 0$, indicating that the isotropic dissipation rate $\overline{\varepsilon}_{iso}$ is a good surrogate of $\bar{\varepsilon}$, as it was also observed in the intermediate wake of SC [6].

4 Conclusions

The TKE transport equation (Eq. (1)) is tested in the far field of a SC wake and compared with that in a CC wake. It is found that there are two significant differences between SC and CC wakes associated with the TKE budget:

(1) the physical mechanism for transporting TKE along the axis is different from that in a CC wake. Indeed, for the SC wake, the $\overline{q^2}$ diffusion term is negligible compared to advection term along the axis and the advection and energy dissipation terms dominate the budget. In the CC wake, aside from the advection and energy dissipation terms, the diffusion term also contributes significantly to the budget.

(2) at the centerline of the wake, the gain of the energy due to the contributions from the advection and diffusion terms is equal to the loss due to the isotropic dissipation. This indicates that the isotropic dissipation rate $\overline{\varepsilon}_{iso}$ is a good surrogate of the mean TKE dissipation rate $\bar{\varepsilon}$.

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